Towards an Improved High Dispersion Ripple Correction

The current SIPS high dispersion ripple correction formula is a parameterized sinc function of the form

$$\frac{\sin^2 \pi X}{(\pi X)^2} \quad (1 + a\pi^2 X^2)$$

where

$$X = \text{MIN} \begin{cases} m\ (\lambda/\ \lambda_c - 1) \\ 2.61/\pi \end{cases}$$

m being the order number, and λ_c the central wavelength corresponding to the peak of the blaze.

The parabolic factor was introduced when the observed blaze function was found to be broader than the theoretical sinc. In NASA IUE Newsletter, 14 I. Ahmad suggested that the SWP ripple correction could be better represented by a sinc function with no parabolic correction of the form

$$\frac{\sin^2 \pi\alpha X}{(\pi\alpha X)^2}$$

where $\alpha = 0.85$. This function appeared to fit the ends of the orders better and had the aesthetic advantage of introducing the parameterization directly into the sinc function.

In an attempt to justify the Ahmad fit, an investigation was begun to derive a more complete theoretical form of the diffraction envelope produced by a perfect plane blazed grating used in high orders. The result is that, with a slight change in the definition of X, the Ahmad parameterization is the appropriate functional form that should be used for the ripple correction, the parameter α being dependent upon the profile of the grating grooves.

In addition an effort was made to find a theoretical cause for the apparent variation of the grating constant $K = m\lambda_c$ as documented, for example, by Beeckmans and Penston (Three-Agency Meeting Report, 1979). With this simple theory, no explanation could be found.

The following sections discuss the derivation of the blaze function in wavelength space, the least-squares fitting of the sinc function to IUE standard stars, and the limits of using this function to connect for the ripple.

a. Derivation of the Blaze Function

The blaze function of a plane grating used in high orders can be adequately approximated in scalar theory by the diffraction pattern produced by a single groove facet. Consider the grating profile of figure 1 for a grating with groove frequency 1/d, facet length a and blaze angle γ.

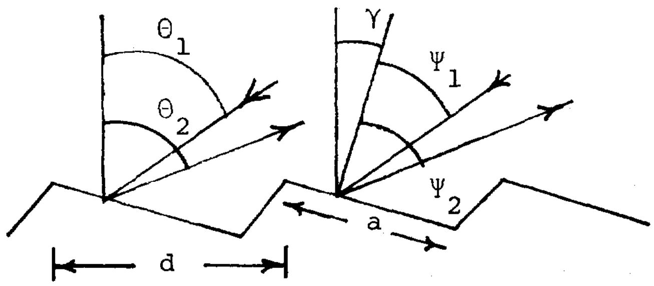


Figure 1

The dispersive properties of the grating are set by the incident and diffracted angles with respect to the grating normal and the groove spacing d through the grating equation

$$m \lambda = d (\sin \Theta_1 + \sin \Theta_2).$$

The blaze pattern is determined by the facet length a and the incident and diffracted angles measured from the facet normal:

$$\sin^2 \pi X / (\pi X)^2 \qquad (1)$$

where

$$X = \frac{a}{\lambda} (\sin \Psi_1 + \sin \Psi_2)$$

Substituting $\Psi_i = \Theta_i - \gamma$, one can show that

$$X = \frac{a}{\lambda} \left[\cos \gamma \left(\frac{m\lambda}{d} \right) - \sin \gamma (\cos \Theta_1 + \cos \Theta_2) \right] .$$

To eliminate the cos Θ_1 term, we note that the maximum of the diffraction envelope occurs where $\Psi_1 = - \Psi_2$, so that at the center of the blaze $\Theta_1 + \Theta_{2c} = 2 \gamma$. From the grating equation we can show that

$$\cos \Theta_1 = \cot \gamma \, (m \lambda_c / d) - \cos \Theta_{2c}$$

and thus

$$X = \frac{a}{\lambda} \left\{ \cos \gamma \left(\frac{m}{d}\right) (\lambda - \lambda_c) - \sin \gamma \, (\cos \Theta_2 - \cos \Theta_{2c}) \right\} .$$

Differentiating the grating equation with respect to Θ_2 yields

$$\cos \Theta_2 = \frac{m}{d} \frac{d\lambda}{d\Theta_2}$$

so we find that the argument for the sinc function is of the form

$$X = m \cos \gamma \left(\frac{a}{d}\right) \frac{1}{\lambda} \left\{ (\lambda - \lambda_c) - \tan \gamma \left(\frac{d\lambda}{d\Theta_2} - \frac{d\lambda}{d\Theta_{2c}} \right) \right\} \qquad (2)$$

Note that this is similar to the Ahmad form with $\alpha = \cos \gamma \, (a/d)$, i.e., determined completely by the grating profile. A term also arises due to the change in angular dispersion across the orders.

Since the IUE echelles are used nearly in Littrow mode, we can eliminate the angular dispersion factor in the following manner. We write the angular dispersion in the form

$$\lambda \frac{d\Theta_2}{d\lambda} = \frac{\sin \Theta_1 + \sin \Theta_2}{\cos \Theta_2} .$$

In near Littrow mode, $\Theta_1 \simeq \Theta_2 \simeq \gamma$ and the dispersion can be approximated by

$$\lambda \frac{d\Theta_2}{d\lambda} \cong 2 \tan \gamma \ . \qquad (3)$$

We then arrive with

$$X = 1/2 \ m \cos \gamma \left(\frac{a}{d}\right) [1 - \lambda_c/\lambda] \qquad (4)$$

which is of the Ahmad form with the modification that the term in brackets in (2) has been divided by λ rather than λ_c.

Since the Ahmad α merely sets the width of the blaze function and cannot explain the observed variation of K, a more complex functional form of X was also examined that included the dispersion term in (2). Writing this term in the form

$$-\tan \gamma \frac{d\lambda}{d\Theta_{2c}} \left(\frac{d\lambda}{d\Theta_2} \Big/ \frac{d\lambda}{d\Theta_{2c}} - 1 \right) \qquad (5)$$

and assuming a constant camera focal length for each wavelength along an order so that the ratio of angular to linear dispersions is constant, equations (2), (3) and (5) yield

$$X = m\,\alpha \left\{ 1 - \lambda_c/\lambda \left[1 + 0.5 \left(\frac{d\lambda}{dx} \Big/ \frac{d\lambda}{dx_c} - 1 \right) \right] \right\} \qquad (6)$$

b. Fits to the Sinc Function

Least-squares fits of equations (1), (4), and (6) were performed on IUE standard stars using the software developed by Ahmad. The fitting routine yields values of the grating constant K, fitting parameter α, and the intensity normalization scale factor as derived for each order. To minimize the effects of noise at the ends of the orders, the first and last 25 points were eliminated from the fitting.

Figure 2 shows a typical fit to the full dispersion sinc form compared to the correction supplied by SIPS. The shape of the two ripple curves both adequately follow the observed blaze function, but because the SIPS correction utilizes a constant K, the peak of the curve is shifted in wavelength.

Figure 3 illustrates a typical variation of K and α with order for both spectrographs from equation (6). While K establishes the wavelength centering, the α parameter determines the width of the ripple and is found to be constant with order except at the weakly exposed highest and lowest orders.

It was found that even the full dispersion fit could not remove the apparent variation of K. Some of the variation can be attributed to camera wavelength sensitivity changes across each order. This affects both the shape and wavelength center of the observed blaze. Figure 4 shows the LWR K values derived by using the low dispersion sensitivity curve as an approximation for the camera sensitivity. As expected, the greatest difference occurs at the lowest and highest orders where the slope of the sensitivity curve is the steepest.

Figure 5 illustrates part of a ripple-corrected LWR spectrum using fits to equation (4) both with and without the low dispersion sensitivity. When ignoring the sensitivity curve, each order is merely flattened so that the ends do not properly overlap with adjacent orders. Including the sensitivity gives the appropriate overlap. Clearly a more detailed formulation incorporating a treatment of the full echelle setup will be necessary to derive accurate flux measurements. Such work is now underway.

Conclusions

The parameterized sinc function as suggested by Ahmad is found to be a more appropriate form for the ripple correction because it can be justified from physical optics. In practice, the current SIPS function differs marginally in shape from the true form; however for both corrections, a varying K factor is necessary to properly align the peak of the blaze pattern to the observed values.

Work has begun to determine a set of K values that can be used to insure that adjacent orders properly overlap. Already there is evidence that the K values change with time and camera temperature due to shifts of the spectral format on the camera faceplates.

As the theoretical blaze derived in this study is appropriate for a single perfect plane grating illuminated in unpolarized light, it is not surprising that the simple sinc form cannot explain all the details of a full echelle system.

Thomas B. Ake

9 September 1981

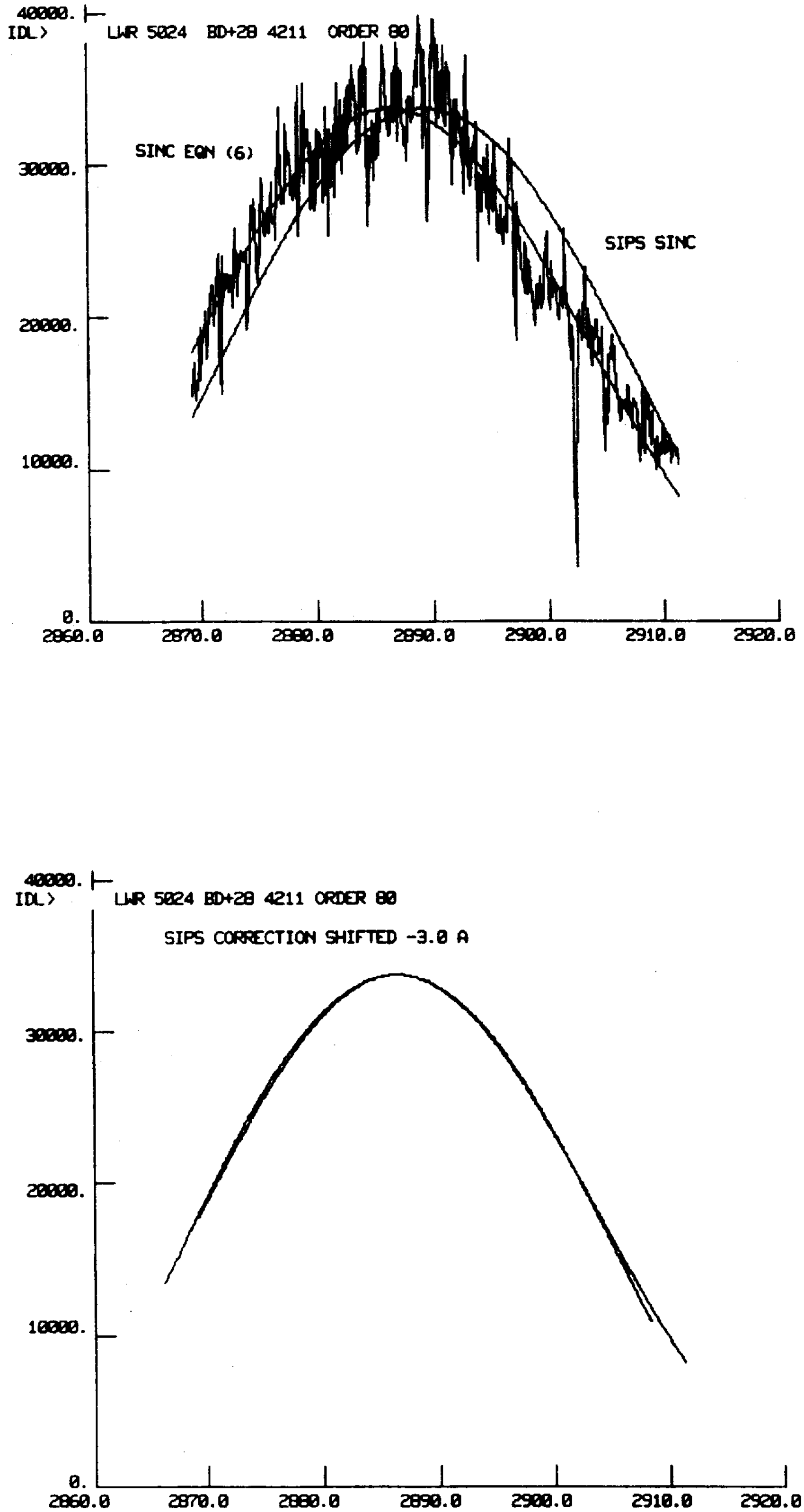
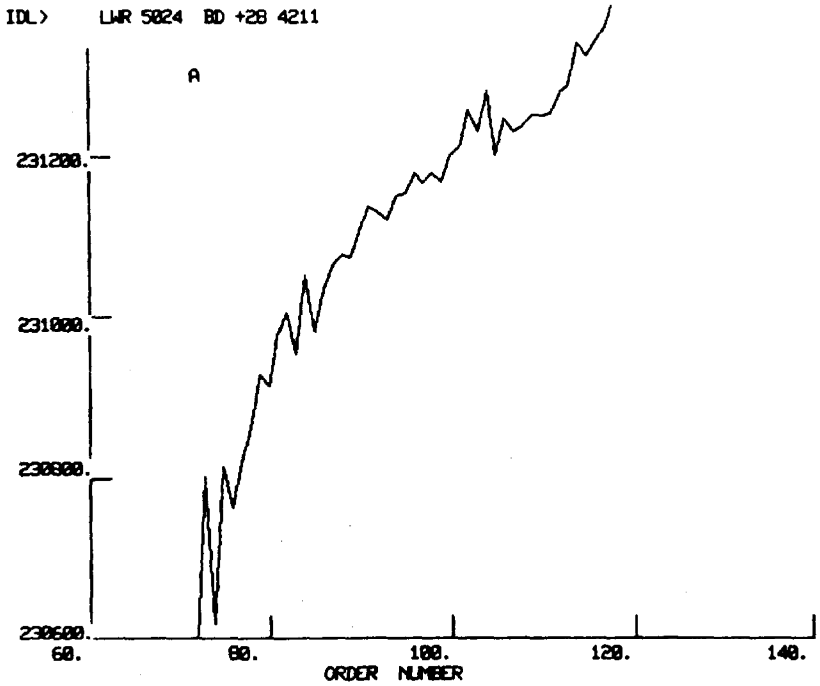


FIGURE 2. SINC AND SIPS RIPPLE CORRECTIONS



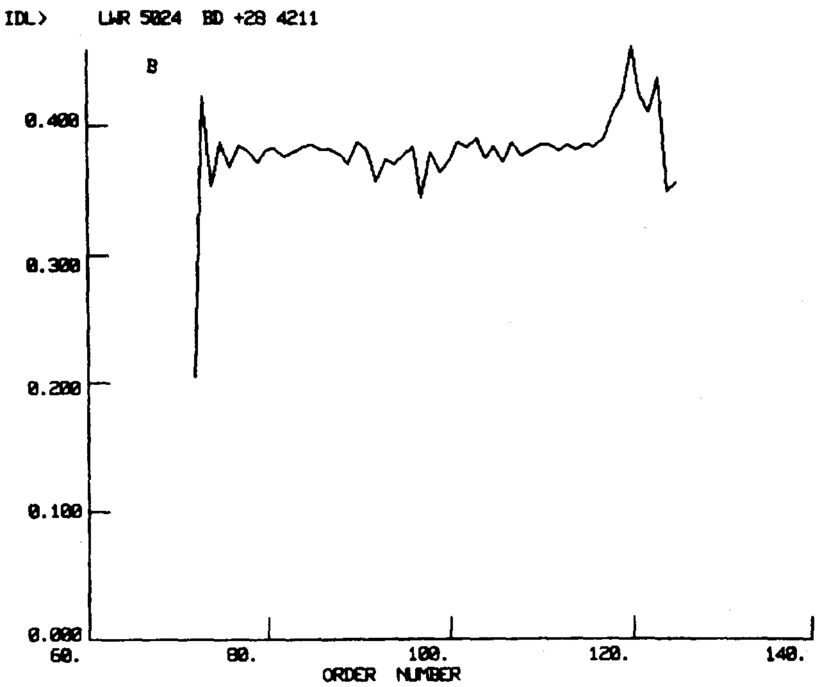
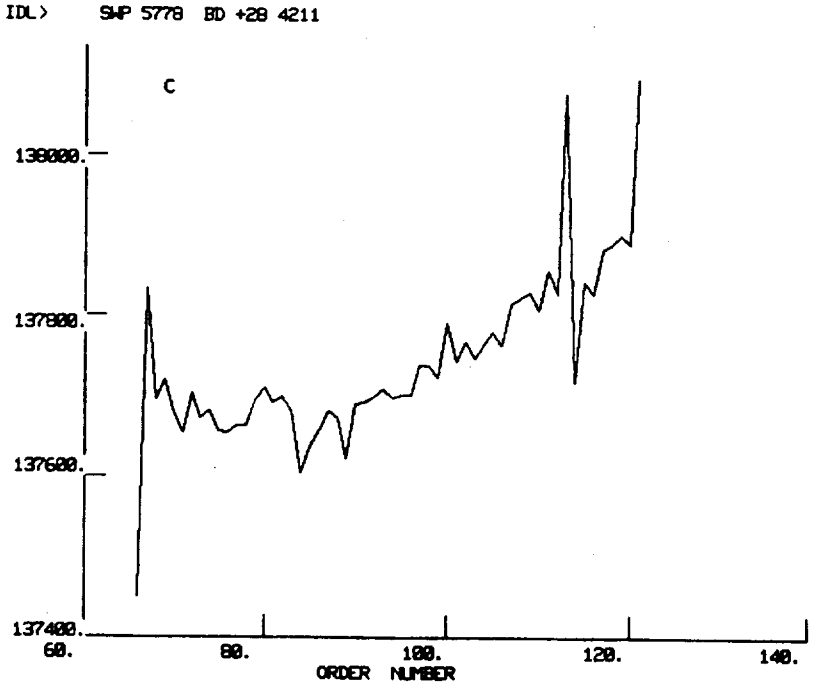


FIGURE 3 A,B LWR 5024 DERIVED K AND ALPHA VALUES FOR EACH ORDER



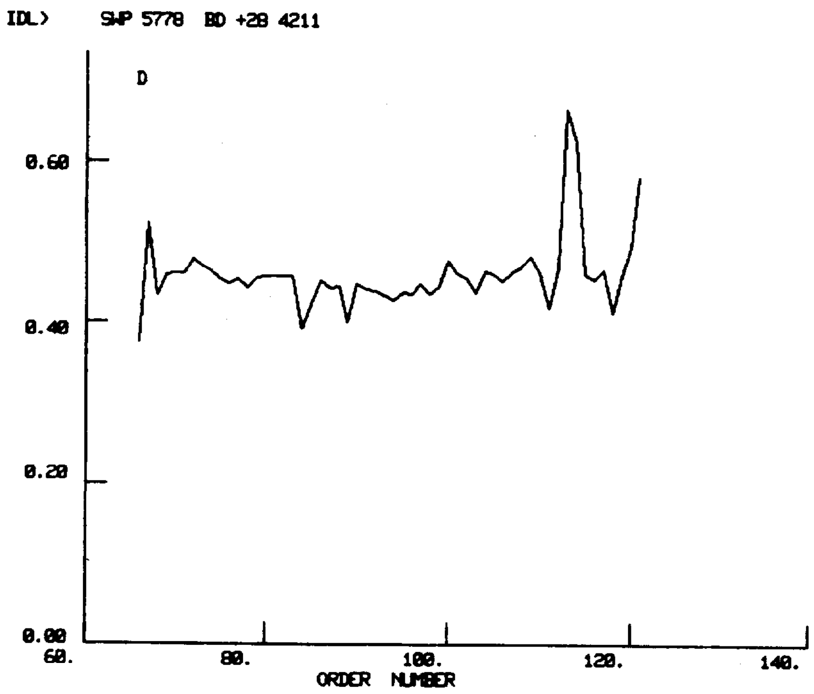
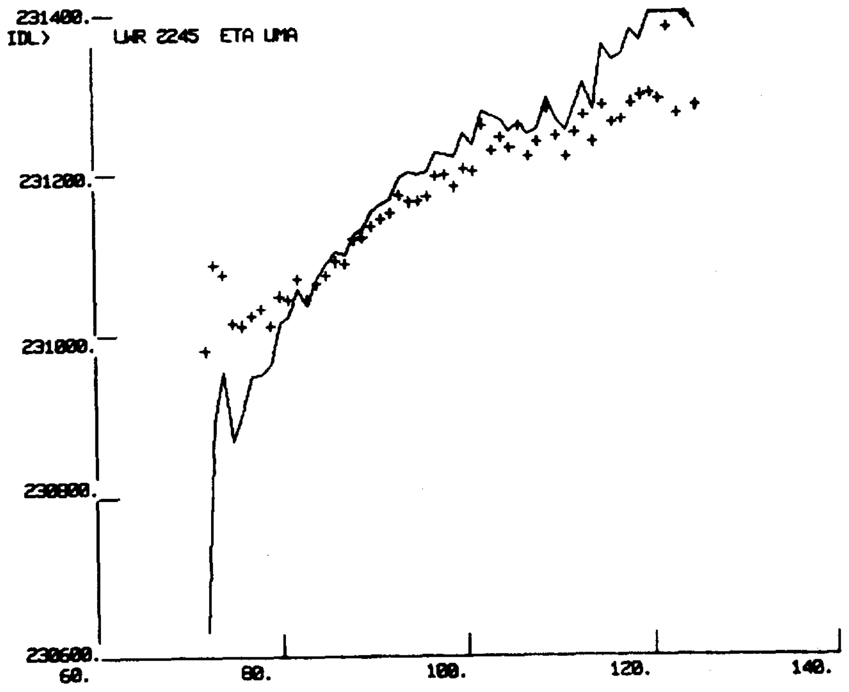


FIGURE 3 C,D SWP 5778 DERIVED K AND ALPHA VALUES FOR EACH ORDER



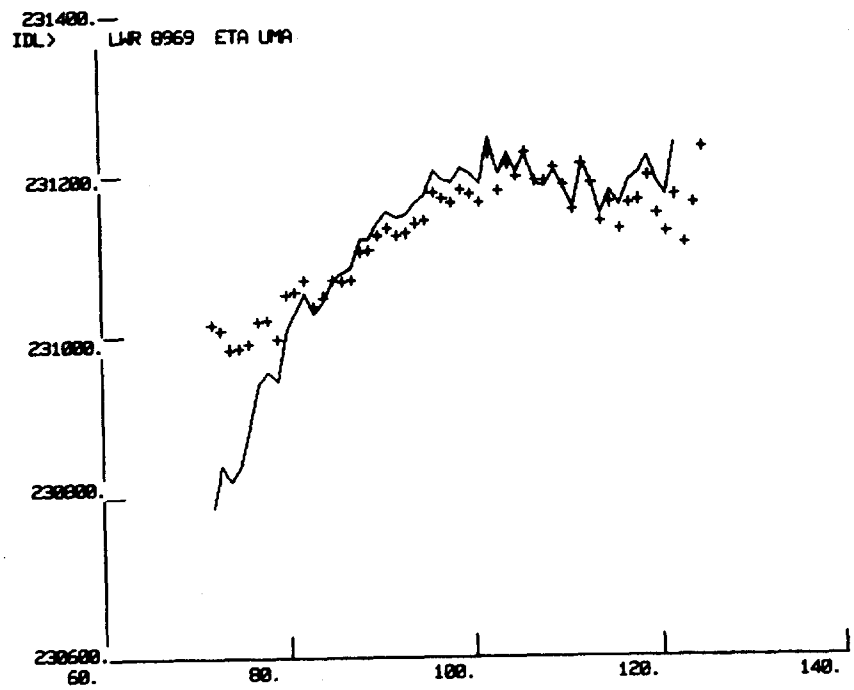
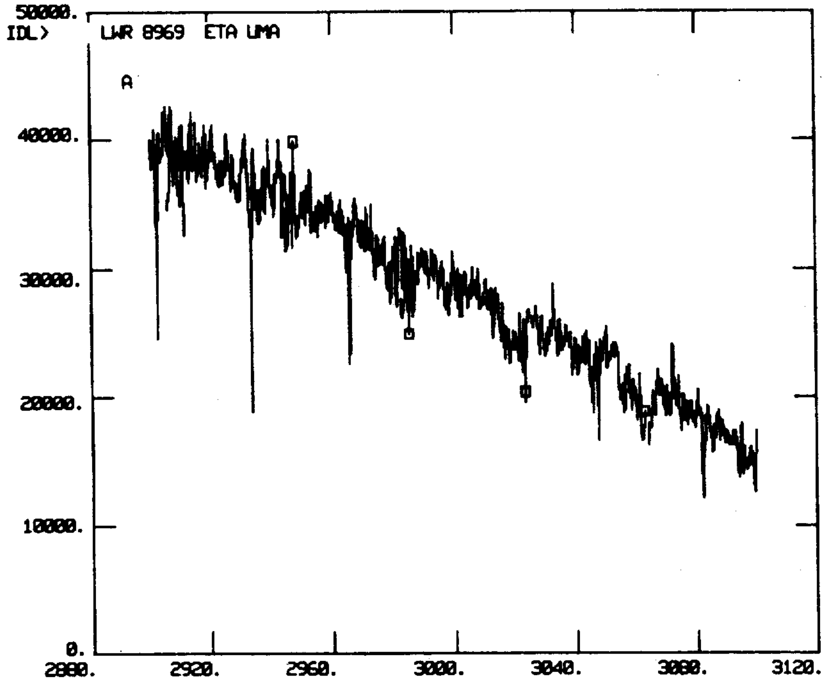


FIGURE 4. LWR 8969 K VALUES DERIVED WITH THE LOW-DISPERSION SENSITIVITY (+) AND WITHOUT (—).



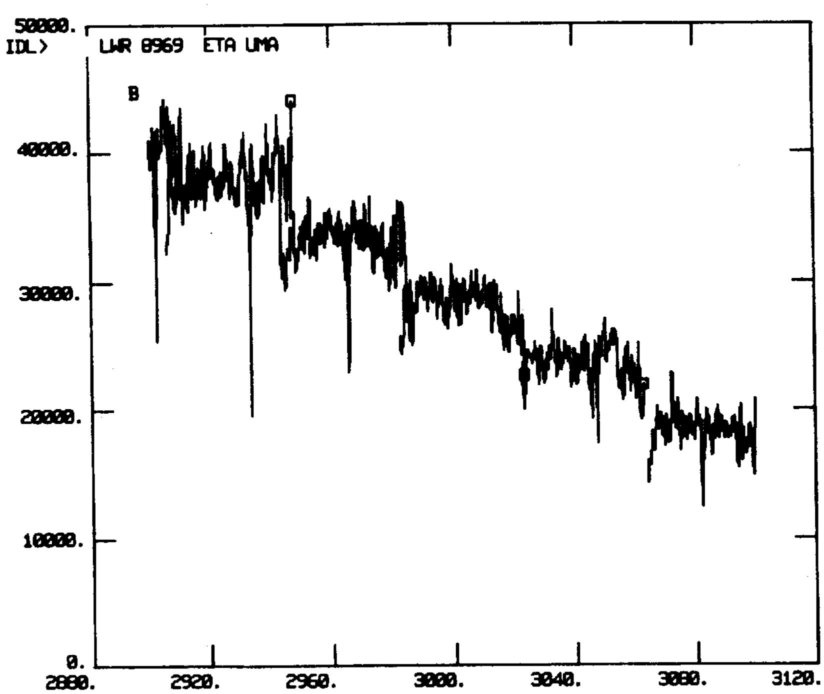


FIGURE 5. LWR 8969 RIPPLE CORRECTED PLOTS. (A) K VALUES DERIVED WITH THE LOW-DISPERSION SENSITIVITY, AND (B) WITHOUT.