Improvements in the Photometric Correction: Non-Linear Interpolation between ITF Levels

RICHARD A. SHAW
Computer Sciences Corporation

I. Introduction

Strange artifacts seen in IUE spectra have on occasion been traced to an inadequate algorithm for photometrically correcting pixels that are nearly saturated, or that have DN values in excess of the highest level of the Intensity Transfer Function (ITF). Those levels of the ITF that lie above 250 DN have been considered unreliable and are currently excluded in standard IUESIPS processing. When applying the photometric correction, DN values lying between the lowest ITF level and the highest level below 250 are converted to FN by linearly interpolating between the adjacent ITF levels. All DN values above the highest reliable ITF level are converted to FN by linearly extrapolating from the last three ITF levels that lie below 250 DN; those DN values below that corresponding to zero FN are computed by extrapolating linearly from the lowest two ITF levels.

Figure 1 illustrates how the ITF is applied in current IUESIPS processing. The dotted line shows the current linear interpolation between ITF levels, and the heavy, solid lines indicate regions of extrapolation. Note the discontinuity of ~ 10 FN at the highest used ITF level, as well as its obvious departure from the smoothly-varying curve that would result by not excluding the next point at 255 DN (the highest point is not distinct). Although there is undoubtedly some uncertainty in determining the precise FN value where any particular pixel saturates, points above 250 DN are not wholly unreliable. The analysis presented below will in part reexamine the reliability of nearly saturated pixels in the context of a fit or superior interpolation for the ITF curves *via* a smooth and slowly-varying calculable function. The aim will be to derive a more accurate photometric correction, particularly for nearly saturated pixels.

II. Interpolation Schemes

If the ITF levels are accurately determined, then *interpolation* between and extrapolation beyond them, rather than *fitting* a function through them, is the most appropriate means of applying the photometric correction. Three different interpolation schemes were considered (aside from linear) for this purpose. Figure 2a illustrates a cubic spline fit for one pixel near the center of the LWP ITF. The interpolation is extended beyond the endpoints by linear extrapolation from the nearest two ITF

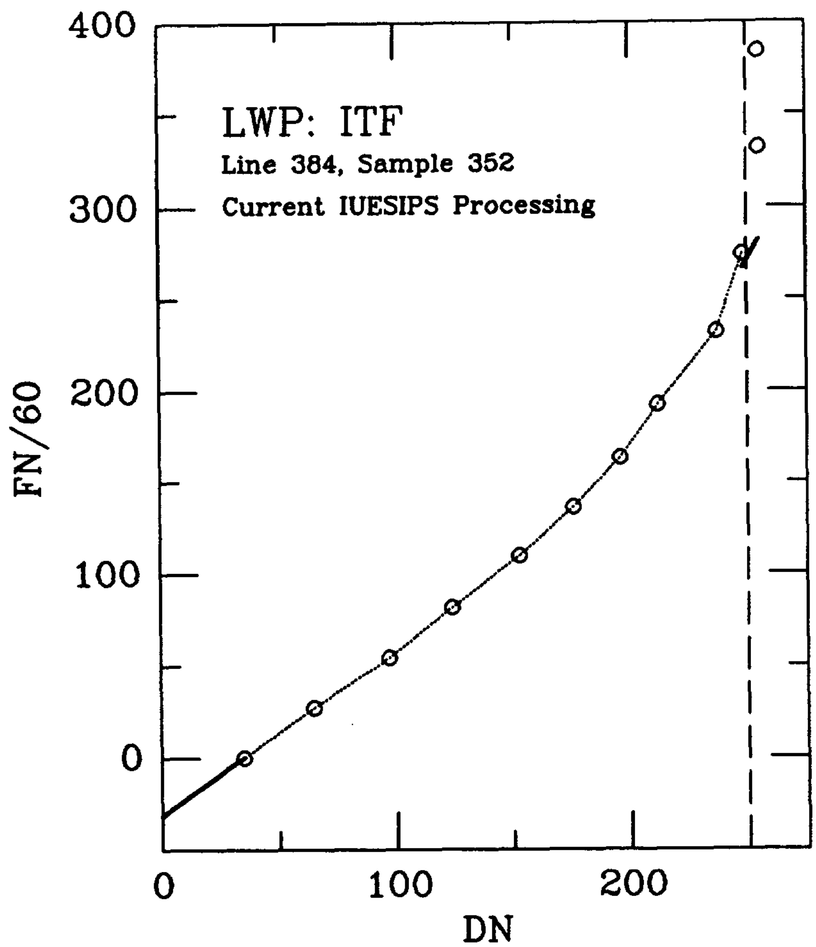

FIG. 1—Example ITF curve showing regions of interpolation (*dotted line segments*) between, and extrapolation (*heavy solid lines*) beyond, the ITF levels (*open symbols*) in current IUESIPS processing. Points lying to the right of the *dashed line* have been considered unreliable.

levels. Note that the fit is very poor where the slope changes abruptly, in the sense that the interpolated values are alternately higher, and then lower than what appears reasonable by inspection. This "ringing" characterizes the spline fits for all of the considered pixels, even when the highest level is excluded from the interpolation (as was done here) to avoid the extreme slope changes at the high end.

Rational functions (RFs), or ratios of polynomials, were also considered in the hope of avoiding the ringing that typifies splines and simple interpolating polynomials. Figure 2b illustrates a RF interpolation for the same pixel, but with no levels excluded. Unfortunately, this sort of function suffers discontinuities at various places along the curves, even where the curves are most linear. Evidently, they result from the incomplete cancellation of the polynomial in the denominator approaching zero as that in the numerator approaches infinity. In an effort to circumvent this problem,

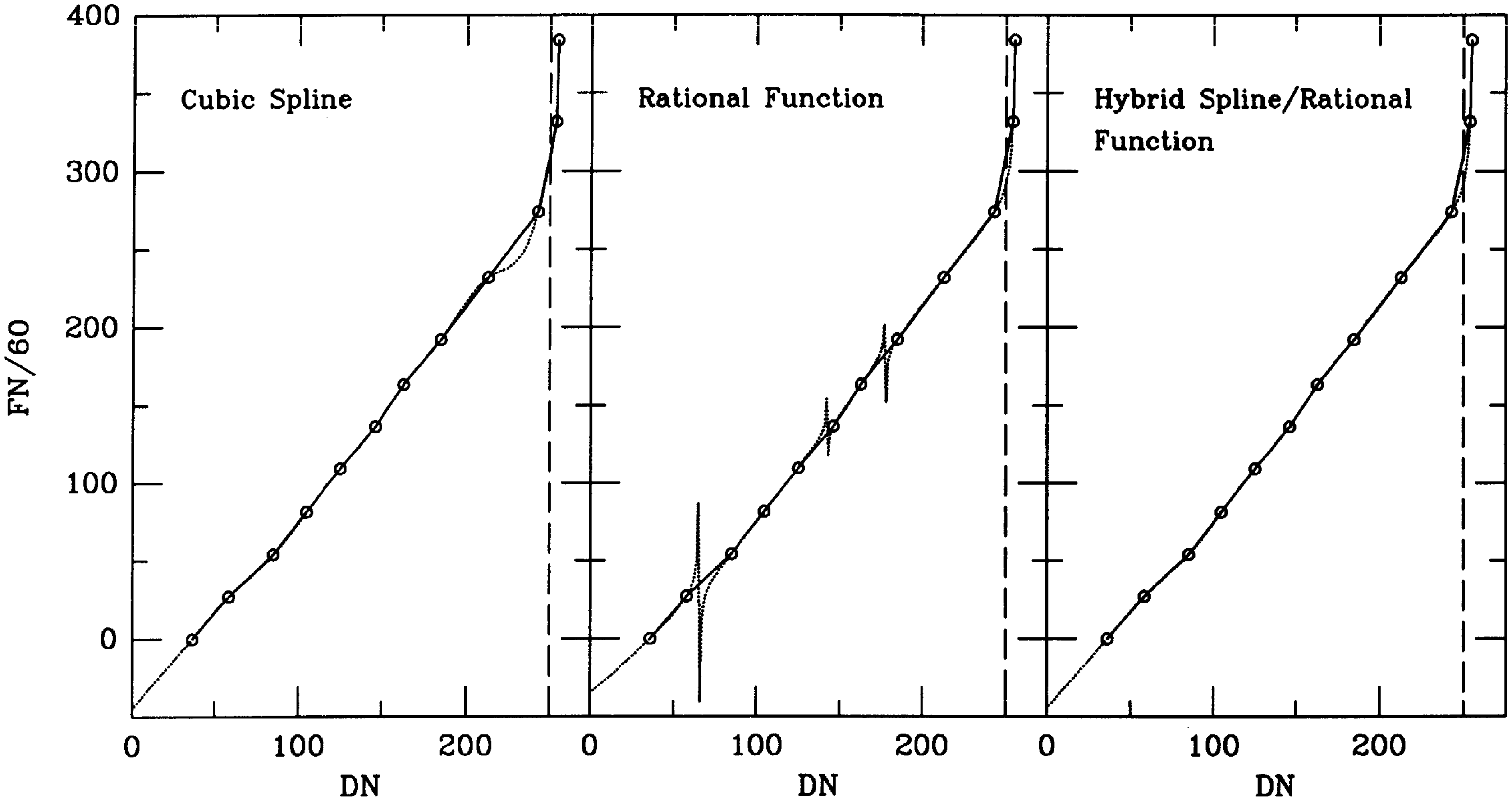

FIG. 2—ITF curve for `line` 384, `sample` 348, for the LWP camera showing three different interpolation schemes (*dotted curves*) between ITF levels. Shown are cubic spline (*left*), rational function (*center*), and a hybrid spline/rational function (*right*); simple linear interpolation (*solid line segments*) is shown on each plot for comparison.

RFs of lower order were fit to subsets of successive points, and the segments were joined piecewise to describe the entire ITF curve for a given pixel. Unfortunately, this approach does not guarantee against discontinuities. The RF interpolation is usually very good where the slopes become extreme, however, which is precisely where the cubic spline fails. Figure 2c shows a hybrid of the two functions, where the RF was used for the highest five ITF levels, and the cubic spline elsewhere. Although discontinuities occasionally appear in these restricted RF interpolations, and the slope discontininuity where the functions join sometimes results in poor interpolations, they were very good for $\sim 80\%$ of the ITF curves attempted.

The major drawback to applying the above interpolation schemes lies with the extra computer time that is required to do the calculations for $\sim 5\times10^6$ pixels per high-dispersion IUE image. The problem could be avoided if the FN values corresponding to all 256 possible DN values were calculated in advance and stored for each pixel, but the resulting ITF files would be huge and possibly cumbersome to use. As an alternative, Chebyshev polynomials were fit to the ITF curves since they have the virtue of yielding the best approximation to a function in the least number of terms. Provided that number of terms is low enough, the coefficients of the best fit for each pixel could be stored in considerably less space. As discussed by Press, *et al.* (1986), the function to be approximated must be calculable at the n zeros of each of the terms used in the approximation; these values were obtained using the hybrid interpolation function discussed above. The fit was performed with 50 terms in order to ensure very close agreement with the hybrid function, but the fit was evaluated with many fewer terms in order to determine the minimum necessary to yield an acceptable approximation. Figures 3a and 3b show the resulting fits for 32 and 16 terms, respectively. Note that the latter deviates noticeably from the desired function, which is plotted in Fig. 2c. The deviations relative to simple linear interpolation are plotted in Fig. 4 for the Hybrid and the two Chebyshev polynomials. Evidently, the number of terms necessary to approximate the interpolating function to within a few tenths of an FN is not much less than 32. Storing 32 coefficients would consume one-fourth as much disk space of a 256-level ITF file, but would require about 5 times the space of the current ITFs and would still require a calculation (*vis a viś* an array look-up) to obtain the function value.

III. Fitting Functions

Another approach to deriving smooth ITF curves is to fit them with smooth, slowly varying functions that do not necessarily pass through any particular point, as illustrated in Fig. 5 for a particularly well-behaved pixel. Here two apparently discrepant points were excluded from a smooth spline fit to the remaining points. While the resulting curve for this pixel is quite appealing, there is no *a priori* reason to exclude any levels from any ITF curve (except when the pixel saturates) without a detailed knowledge of the noise inherent in the various ITF levels. The noise *was*

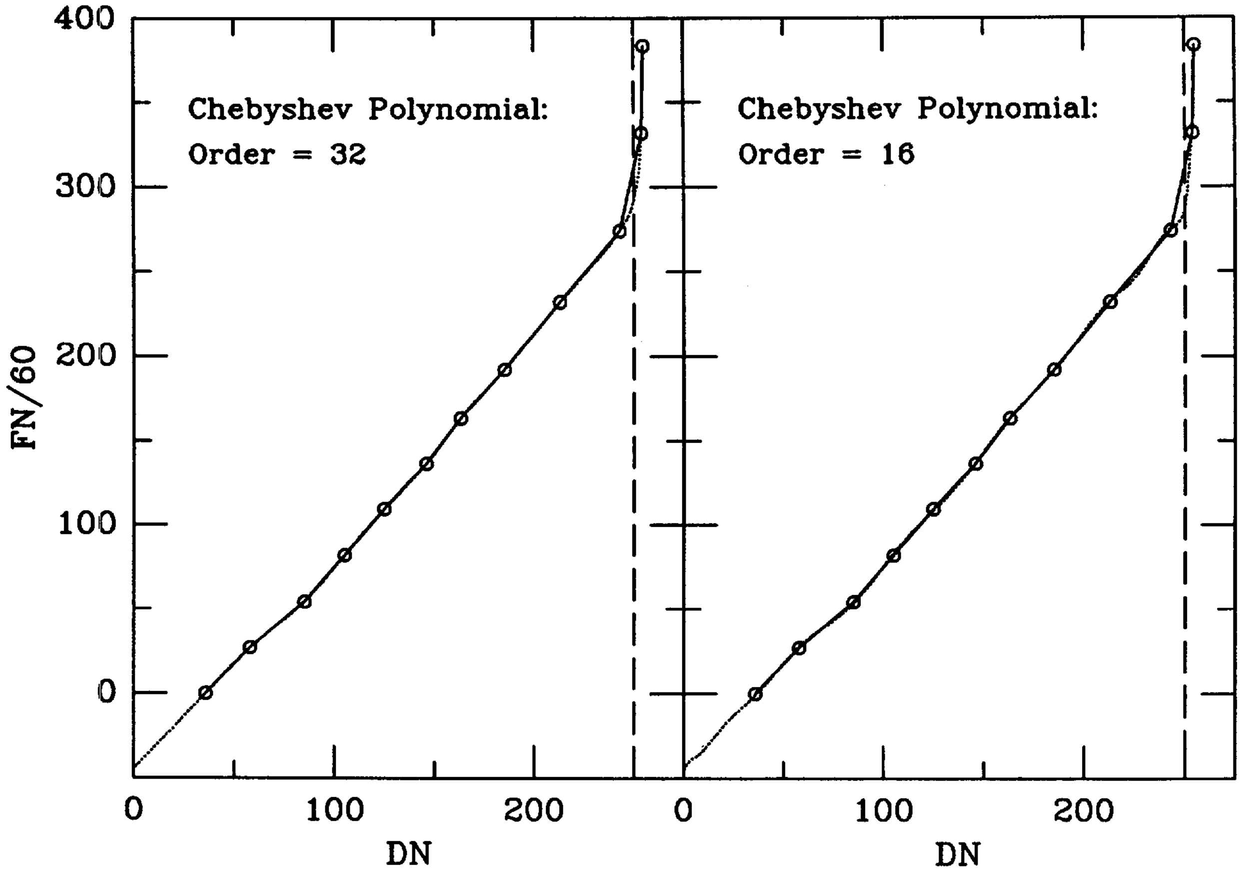

FIG. 3—Same as Fig. 2c, but for Chebyshev polynomial approximations with 32 terms (*a*, at left) and 16 (*b*, at right) terms to the hybrid interpolating function.

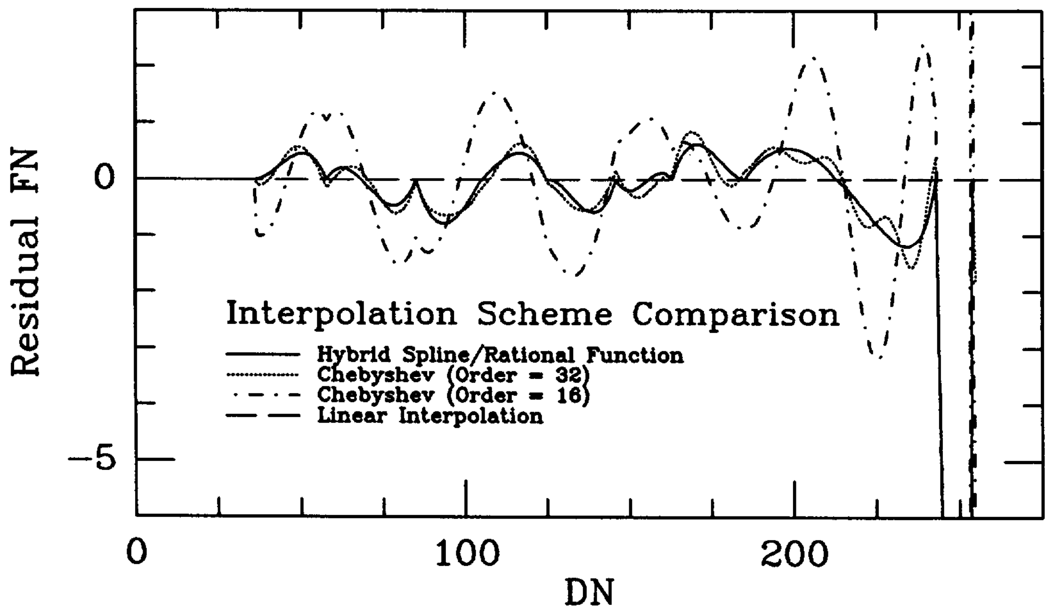

FIG. 4—Difference in FN between various interpolation schemes, as keyed in the legend, and linear interpolation.

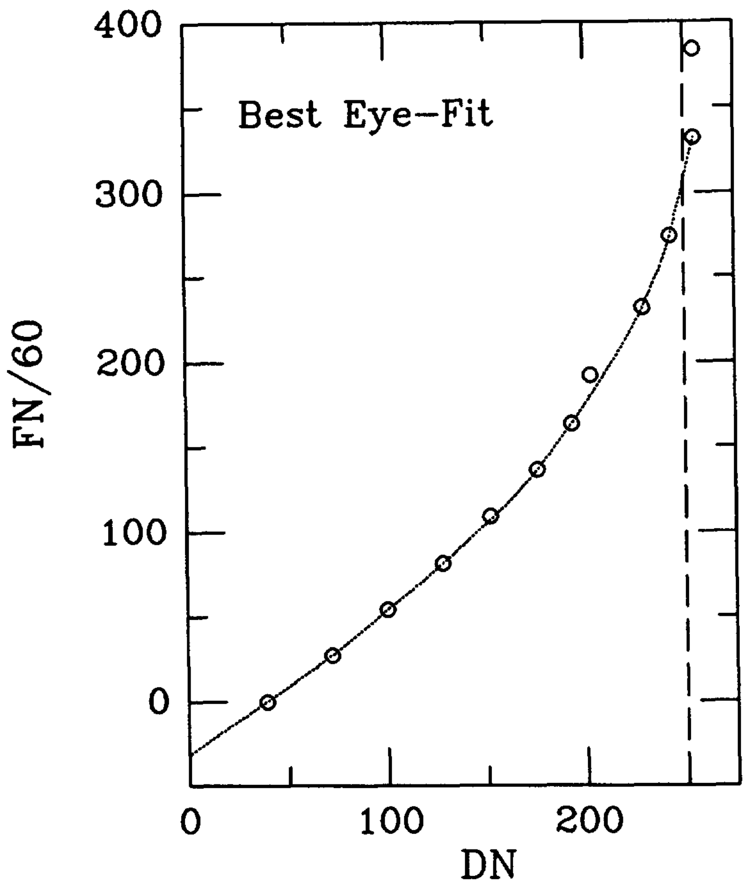

FIG. 5—Fit to a particularly simple ITF curve using a cubic spline and excluding two points that, by inspection, do not seem to fit.

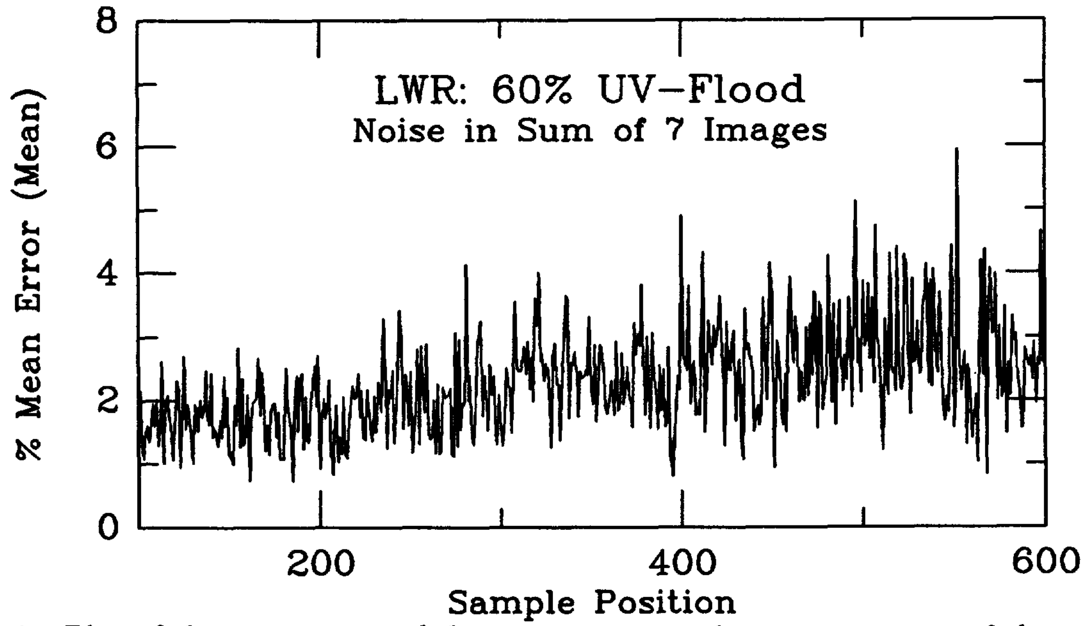

FIG. 6—Plot of the mean error of the mean, expressed as a percentage of the mean DN value, for pixels near the center of this sum of seven 70% UV-Floods. The images were used to construct one level of the LWR ITF.

computed for a portion of the fifth ITF level of the LWR camera by summing the 7 constituent images and calculating the mean and standard deviation for each pixel. Figure 6 shows the mean error of the mean, expressed as a percentage of the mean DN value, for a set of adjacent pixels near the center of the summed image. Although this exposure level probably has the least noise overall in each of the three cameras, by virtue of a larger number of available UV-Floods, it suggests that if formal error bars were placed on the ITF curves presented in this discussion, they would be comparable in size to the symbols used in the figures.

Given a formal uncertainty for each ITF level, it would be possible to construct a weighted fit of a simple function through the levels for each pixel. However, it is unclear at present whether a simple enough function can be found that still adequately characterizes the ITF curves for all pixels. Figure 7 shows a simple polynomial fit to the same curve as in Figs. 2 and 3. Once again, the fit is poorest near sharp changes in slope. It would clearly be best to apply a weighted fit, based upon a knowledge either of the uncertainty in each level or of the true functional form (or "shape") of the ITF curves, if such a thing exists.

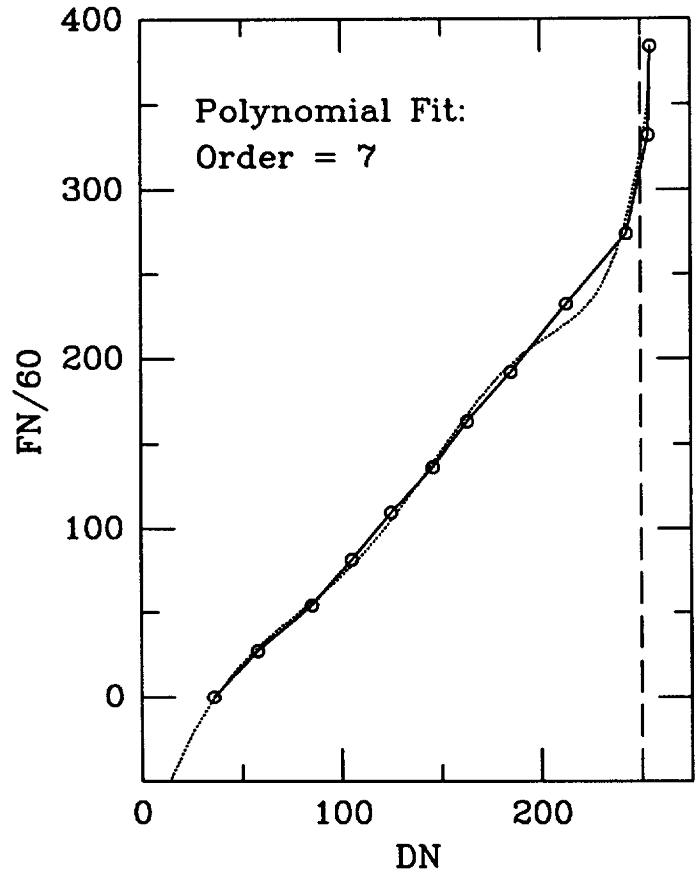

FIG. 7—Seventh-order polynomial fit to the same ITF curve as in Figs. 2–3.

V. Discussion

Clearly, the current method of applying the photometric correction suffers from some inaccuracy, particularly for nearly saturated pixels that have DN values greater than the highest ITF level. Although the current linear interpolation is probably adequate between all but the highest ITF levels, the absence of information about the noise in each level makes it unclear whether a more sophisticated interpolation, or a weighted fit is most appropriate. In the former case, a hybrid of a cubic spline plus a rational function interpolation seems to work very well, but the proper functional form for the latter case is not yet clear. Unfortunately, the noise characteristics for ITFs taken at different epochs are likely to be different, in that the early ITFs are almost certainly noisier because they were constructed from fewer images. Thus the approach taken for the most recent ITFs may not be appropriate for the early epoch ITFs. Either new approach to the photometric correction is more sophisticated and will be more accurate than the current linear interpolation, especially for nearly saturated pixels, but will require either more computer time, or more disk storage space, or both to implement.

Reference

Press, W., Flannery, B., Teukolsky, S., and Vetterling, W. 1986, *Numerical Recipes* (Cambridge: Cambridge University Press).